[2][c]

$$\cosh(x+y) = \frac{e^{x+y} + e^{-(x+y)}}{2}$$

$$\neq \cosh x \cosh y - \sinh x \sinh y$$

$$= \frac{(e^x + e^{-x})}{2}\frac{(e^y + e^{-y})}{2} - \frac{e^x - e^{-x}}{2}\frac{e^y - e^{-y}}{2}$$

$$= \frac{e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y}}{4} - \frac{e^{x+y} - e^{x-y} - e^{-x+y} + e^{-x-y}}{4}$$

$$= \frac{e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y} - e^{x+y} + e^{x-y} + e^{-x+y} - e^{-x-y}}{4}$$

$$= \frac{2e^{x-y} + 2e^{-x+y}}{4}$$

$$= \frac{2(e^{x-y} + e^{-x+y})}{4\ 2}$$

$$= \frac{e^{x-y} + e^{-(x-y)}}{2} = \cosh(x-y)$$

YESTERDAY $\cosh^2 x - \sinh^2 x = 1$

HW : $\cosh 2x = \cosh^2 x + \sinh$

TRIG: $\cos(x+y)$

$= \cos x \cos y - \sin x \sin$

$e^A e^B = e^{A+B}$

$\cosh(x-y) = \cosh x \cosh y - \sinh x \sin$

? $\cosh(x+y) = \cosh x \cosh y + \sinh x \sin$

HW

[3] $\tanh x = \frac{\sinh x}{\cosh x}$ ← DEFINITION

[a] $\tanh x = \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} \cdot \frac{2}{2} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ (1) ↑ DEF'N

$= \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} \cdot \frac{e^x}{e^x}$

$= \frac{e^{2x} - 1}{e^{2x} + 1}$ (2)

$= \frac{e^{2x} - 1}{e^{2x} + 1} \cdot \frac{e^{-2x}}{e^{-2x}}$

$= \frac{1 - e^{-2x}}{1 + e^{-2x}}$ (3)

[b] $\tanh 0 = \frac{\sinh 0}{\cosh 0} = \frac{0}{1} = 0$ ← [0][a]

$\tan 0 = 0$

OR

$\tanh 0 = \frac{e^0 - 1}{e^0 + 1} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

[c] $\tanh(-x) = \dfrac{1-e^{2x}}{1+e^{2x}} = \dfrac{-(e^{2x}-1)}{e^{2x}+1} = -\tanh x$

OR

$\tanh(-x) = \dfrac{\sinh(-x)}{\cosh(-x)} = \dfrac{-\sinh x}{\cosh x} = -\tanh x$

[1][a][b]

[b] DOMAIN OF $\tanh x$

$\tanh x = \dfrac{\sinh x}{\cosh x}$

DOMAIN OF $\sinh x = (-\infty, \infty)$ [0][b]

DOMAIN OF $\cosh x = (-\infty, \infty)$

WHERE IS $\cosh x \neq 0$? $\cosh x = 0$ [0][c]

~~ONLY IF~~ NEVER

$\cosh x \neq 0$ FOR ALL REAL NUMBERS

IE. ON $(-\infty, \infty)$

SO, DOMAIN OF $\tanh x = (-\infty, \infty)$

$\frac{0}{0} \neq 0$

↑

UNDEFIN

SOLVE $\tanh x = 0$

$\dfrac{\sinh x}{\cosh x} = 0$ → $\sinh x = 0$ AND $\cosh x \neq 0$

[0][c]

$x = 0$ AND $x \in (-\infty, \infty)$

$x = 0$

[d] $\operatorname{csch} x = \frac{1}{\sinh x}$

$\operatorname{sech} x = \frac{1}{\cosh x}$

$\coth x = \frac{\cosh x}{\sinh x} = \frac{1}{\tanh x}$

$= \operatorname{ctg} x$

[e] HW

[f] SIMILAR TO [i] [k]-[n]